Row Reduction Worksheet Math 202

Name: ____________________ Section: ____________________

You should be able to do this whole packet in approximately 40 minutes, by working *efficiently* but without rushing.

(1) Solve the system of equations with augmented matrix $\left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -3 & -6 & 6 \\ -2 & 10 & 2 & -6 \end{array}\right]$

$(2 \;\; -6 \;\; 4 \,|\, 0) \leftarrow (-2)r_1$

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -3 & -6 & 6 \\ 0 & 4 & 6 & -6 \end{array}\right] r_3^* = r_3 + (-2)r_1$$

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & 1 & 2 & -2 \\ 0 & 4 & 6 & -6 \end{array}\right] \begin{array}{l} \\ -\tfrac{1}{3} r_2 \\ \\ \end{array}$$

$(0 \;\; -4 \;\; -8 \;\; 8) \leftarrow (-4) r_2$

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & -2 & 2 \end{array}\right] r_3^* = r_3 + (-4) r_2$$

consistent w/ pivot in ea. column
$\Rightarrow$ has unique sln.

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{array}\right] \begin{array}{l} r_1^* = r_1 - r_3 \\ r_2^* = r_2 + r_3 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|c} -1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{array}\right] r_1 - 3r_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\begin{cases} x_1 = 2 \\ x_2 = 0 \\ x_3 = -1 \end{cases} \leftarrow \text{the unique sln.}$$

Name: ______________________________ Section: ______________________________

(2) Solve the system of equations with augmented matrix $\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ -4 & -4 & 6 & -6 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ -4 & -4 & 6 & -6 \end{array}\right]$$

$(4 \quad 8 \quad -4 \quad 0) \leftarrow 2r_1$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ 0 & 4 & 2 & -6 \end{array}\right] r_3 + 2r_1$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

No pivot in col 3
$\Rightarrow \infty$-many slns

$$\sim \left[\begin{array}{ccc|c} 2 & 0 & -4 & 6 \\ 0 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right] r_1 - r_2 \iff \begin{cases} x_1 \quad -2x_3 = 3 \\ x_2 + \frac{1}{2}x_3 = -\frac{3}{2} \\ x_3 \text{ free} \end{cases}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \frac{1}{2}r_1 \\ \frac{1}{4}r_2 \end{array} \iff \begin{cases} x_1 = 3 + 2x_3 \\ x_2 = -\frac{3}{2} - \frac{1}{2}x_3 \\ x_3 \text{ free} \end{cases}$$

(3) Solve the system of equations with augmented matrix $\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ -4 & -4 & 6 & 0 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ -4 & -4 & 6 & 0 \end{array}\right]$$

$(4 \quad 8 \quad -4 \mid 0) \; 2r_1$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 4 & 2 & 0 \end{array}\right] r_3 + 2r_1$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & -6 \end{array}\right] r_3 - r_2$$

$\boxed{\underline{\text{No}} \text{ solution by theorem 2}}$

Row Reduction Worksheet Math 202

Name: ______________________________ Section: ______________________________

(4) Give an example of the augmented matrix of a system of 3 equations in 3 variables ...
(*Hint:* you can make the systems as simple as you like.)

rows coeff col's

(a) with a unique solution

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

(b) with no solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

(c) with infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

No pivot

(5) Give an example of the augmented matrix of a system of 2 equations in 3 variables ...
(*Hint:* you can make the systems as simple as you like.)

rows coeff col's

(a) with no solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(b) with infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 2 \end{array}\right]$$

No pivot

(6) Use Theorem 2 to prove that you cannot write a system of 2 equations in 3 variables that has a unique solution. (*Hint:* your argument must consider all relevant reduced-echelon form matrices.)

$P \Leftrightarrow Q$ By theorem 2
unique solution $\Leftrightarrow$ has pivot in each coeff column

But

$\neg Q$
$$\left[\begin{array}{ccc|c} * & * & * & * \\ * & * & * & * \end{array}\right]$$

You cannot fit 3 pivots in two rows.

$\neg P$ Therefore the system cannot have a unique solution

Name: ______________________ Section: ______________________

7. Find the general solution (in parametric form) for a system whose augmented matrix is row equivalent to

$$\left[\begin{array}{ccccc|c} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{array}\right] \leftarrow$$

NOTE: ① already in echelon form

② no $[0 \cdots 0 \mid \blacksquare]$
$\Rightarrow$ has a solution by theorem 2.

③ some coeff columns lack a pivot
$\Rightarrow$ has ∞-many solutions

work $\boxed{\text{bottom}}$ to $\boxed{\text{top}}$ (Back sub

$$\sim \left[\begin{array}{ccccc|c} 1 & 6 & 2 & -5 & 0 & 10 \\ 0 & 0 & 2 & -8 & 0 & 10 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{array}\right] \begin{array}{l} R_1+2R_3 \\ R_2+R_3 \\ \end{array}$$

$(0\ 0\ -2\ 8\ 0 \mid -10)\ -R_2$

$$\sim \left[\begin{array}{ccccc|c} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & -8 & 0 & 10 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{array}\right] R_1-R_2$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{array}\right] \tfrac{1}{2}R_2$$

$\Leftarrow$ Reduced echelon form

x_1, x_3, x_5 bound (have pivots)

x_2, x_4 free (no pivots.

The original system is equivalent to

$$\begin{cases} x_1 + 6x_2 \quad + 3x_4 = 0 \\ x_2 \text{ free} \\ x_3 - 4x_4 = 5 \\ x_4 \text{ free} \\ x_5 = 17 \end{cases}$$

$$\boxed{\text{General solution set} = \begin{cases} x_1 = -6x_2 - 3x_4 \\ x_2 \text{ free} \\ x_3 = 5 + 4x_4 \\ x_4 \text{ free} \\ x_5 = 17 \end{cases}}$$